Isaac Physics Skills

How to Solve Physics Problems on Isaac Physics & Beyond!

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How to solve Physics Problems

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Acknowledgements

We are the sum total of our experiences, and so this book is the sum total of my teachers. Some of them will be found in my approach to physics, some of them in my approach to teaching, and some of them in the life that led me here. Thank you all.

More recently, I wouldn't have been able to do this without all of my colleagues at Isaac Physics. Thanks should go to Mark Warner and Lisa Jardine-Wright for allowing me to take this idea and run with it. Thanks to Robin Hughes for being an invaluable officemate, as well as running his keen eye over of the initial draft of this book. Also, thanks to fellow content writer Nicki Humphrey-Baker - it's been a pleasure working with you. To all the rest of the team - thanks for a few great years working on the project. I miss it already.

To the authors of the previous Isaac Physics books, thanks for showing me how it's done. The typesetters of those books, likewise - I have liberally borrowed from your work.

So many thanks and so much gratitude to my parents, my first and best teachers.

And of course, to my wife Jasmin, for putting up with me disappearing off to write this book. You make my life amazing in so many ways.

Contents

Introduction

You have some values. They go in equations. Sometimes you have to rearrange the equations. Sometimes you don't. Learn the equations, learn how to put the numbers in, learn a few facts about the topic and you're done. Right?

Until you come across a situation that's a bit more complicated than that. Maybe none of the equations you know quite fit, or you've got two different accelerations and don't know which to use. Whether it's a question that's been set to you by your teacher or one that you've come across on your own on Isaac Physics, you know that you're missing something somewhere.

That's where this book comes in. It isn't designed to teach you what we mean by momentum, or what a wave is. Instead, it aims to help you get a deeper understanding of the physics you've learnt by actively using that knowledge to solve problems. I hope that by doing this you'll gain insight as to why all of the ideas that you've learnt about are useful, and start to see physics as more of a unified whole.

After short chapters on general problem solving and using the hints on Isaac Physics, the majority of the book will focus on worked examples of problems which loosely follow the Isaac Physics style (particularly those problems in levels 2 to 5) but have each been written to highlight a particular concept or method that can be useful for similar parts of other problems.

You may find it useful to work through the problems inside this book while reading through it. We recommend using a pen/pencil and paper—technology is excellent, but writing and drawing on paper may help you to remember more of what you've done. Good luck.

1.1 General Problem Solving

When you first come across a new physics problem, the temptation can be to get stuck straight in with the equations. A mass here, a velocity there, and you're away calculating kinetic energy. Or should you be working out the momentum? Or possibly you are looking for an acceleration, and the only reason you've got a velocity is because one of the forces relied on the velocity. It's time to take a step back.

First of all, **read the question**. All of it. I wish I didn't have to say that, but I've come across too many students missing out on crucial marks in an exam due to having misread a question or having not actually finished reading it at all. As you go along, it can be useful to consider what some of the words mean. Words like "light' or "ideal" have particular meanings in these physics problems—you can see more of these at the back of the book. You should also be considering what physical concepts you might need to use to solve the problem—reading the entire question rather than just a small part of it can help here, as the way the question is written can point towards particular ways of solving the problem.

The next step is to extract all of the **useful information** from the question. What is the question asking for? What information does it give you? Students are often told to underline or highlight the key words in a question. This is a good method, provided you know what the key words are. As well as these words, are there any physical quantities given in the question? These could be given as a numerical value with a unit, symbolically, as a description in text (look out for words like “stationary” or “at rest”) or as part of a relationship between two quantities (“the pulling force is twice the magnitude of the weight of the block”).

To pull all this information together, it’s generally a good idea to draw at least one **diagram**. While there are a few questions where the situation is simple enough not to need a diagram or (even more rarely) too complicated or too abstract for a diagram to be sensible, these are far outnumbered by questions where a diagram would be useful. The diagram doesn’t have to be a work of art, just understandable. This means that while it’s totally acceptable to draw a car or a lorry as a blob, it should be a labelled blob. Drawing diagrams bigger than they need to be is better than drawing them too small and making them hard to read. Where you use letters to represent quantities, it’s generally a good idea to use an “obvious” letter (like m for mass), but it’s more important to make sure that you can’t confuse different things. This means that you should follow these guidelines:

- Use different symbols (which can be letters with subscripts, such as m_e for the mass of an electron) for each quantity that may be involved;
- Avoid writing numerical values directly on the diagram—add a legend next to it if needs be;
- Make sure to write down next to the diagram what each letter means if the diagram doesn’t make it clear what all the letters refer to;
- Make sure that letters are clearly written and placed on the diagram so you can’t confuse what the letter is or which part of the diagram it refers to.

If you find that you can’t fit all of the useful information you have on one diagram, then draw multiple diagrams. You may want to draw diagrams showing specific objects or sets of objects or looking at the same object from different angles. Sometimes you might draw forces on one diagram and distances on another. Whatever you decide, you should be thinking about making sure that your diagram is as clear as it can be. It is there to help you with solving the problem, as well as helping anyone else who is going to read your solution to understand your work.

Now you’re nearly ready to start manipulating equations. While by this point you probably won’t be able to look at the problem and work out what needs to be done to solve it, you should hopefully be able to identify some useful things that you can try. With this loose “plan” in mind, gather the equations that you think might be useful and start working through the problem. Don’t be afraid to go back and try something else if you are struggling. One of the key skills of "problem solving" is recognising when something you are trying is not going to work and moving to another method instead. Make sure that you don't give up at the first sign of trouble though. The problems on Isaac Physics are designed to stretch you, and so you are likely to spend a fair amount of time feeling uncertain about what to do next, or even frustrated. Feel free to go back to earlier sections in this guide too. Is there a piece of information that you have now found that you need which may be given by the text of the question? Has your working demonstrated that you've drawn something incorrectly on your diagram and you need to redraw it? Is there another equation which you can use to take some quantities you know and turn them into a quantity that you're looking for? Might you even need several steps using different equations to do that?

These steps are often easier said than done, but with practice you will hopefully be solving problems with a lot more confidence.

1.2 Using the Hints on Isaac Physics

Sometimes people find the hints on Isaac Physics a little weird. They ask why some of the hints are just the information from the question again and then that same information added to a diagram. What they are missing is that most of the hints are designed not just to help you answer the problem that you are on, but also to get into good habits for solving more problems in the future. We hope that as you practise answering problem-solving questions you will come to use the hints less and less.

The hints for most of the questions follow the same structure, with 5 distinct types of hint:

1. Glossary and Concepts
2. Useful Information, Information Calculated Previously and Information Assumed
3. Diagram
4. Useful Equations
5. Hint Video

Some questions may omit one or more of these hint types if they aren't particularly relevant to that question. In Level 6 questions, which go beyond the A-level specifications, we also often do not use this hint structure, instead preferring to give a short hint to how to start to solve the problem or a non-obvious "trick" that you may need either to solve the problem or to make it easier. This guide will focus on the 5 standard hints.

Hint 1 - Glossary and Concepts

This hint is here to help you get started thinking about what branches of physics you may need to solve the problem. You may also find this useful to check that you've covered the appropriate concepts in your studies so far to avoid the frustration of working on a problem before you are well equipped to deal with it.

The words in the glossary are terms which have particular meanings in physics. Some of them, such as "centripetal", are terms which you may not have come across in everyday life but are regularly used in physics to describe a particular concept. Other words in this section describe the assumptions and approximations that are made in a particular problem in order to make it possible to solve without unreasonable levels of calculation. For example, surfaces are often described as "frictionless", even though in the real world there would be a small amount of friction between that surface and another surface moving relative to it. In a physics problem this word means that the magnitude of any frictional force that arises is sufficiently small that by ignoring it we can get a good approximation to the real situation which may be refined further in the future.

All of the concepts given link to a Concept Page on Isaac Physics. These pages are written as helpful reminders for people who have previously studied these concepts, not as a way of learning about a concept for the first time. The top of the page will generally contain a brief description of that concept along with one or more related equations. The sections lower down the page will elaborate on particular aspects of that concept, and the levels given on each section give an idea as to which level of question they may be required for. In some cases the links from the hint go directly to a section of the Concept Page that will be particularly useful for that question, but you may also need to read other sections on that Concept Page.

Hint 2 - Useful Information, Information Calculated Previously and Information Assumed

The second hint is the one that confuses the most people. Why is it just re-stating information given in the question? The answer is that a common problem we've seen when students get stuck with a question is that they haven't managed to extract a key piece of information from the question. This happens for a number of different reasons:

- They've failed to realise that a particular bit of text is giving them that information (saying something is massless, for example);
- They haven't realised that although a value isn't given for a piece of information, it can still be used as a symbol that may cancel out;
- They've just missed it when reading through the question.

Use this hint to check that you've got everything you need to tackle the problem.

Hint 3 - Diagram

As described in the last chapter, drawing a diagram is a key step towards solving most problems. This hint is designed to offer "faded scaffolding": at lower levels it will be a full diagram, ready to use, while at higher levels the diagram may not be drawn in the best way or may be missing some key information. Sometimes at higher levels a diagram is not given—this is because by this point you should be in the habit of drawing one yourself.

Even if there's already a diagram here, it's a good idea to draw and label your own so that you are confident about what all of the terms mean and that you have absorbed all of the information given in the question.

Hint 4 - Useful Equations

This section is pretty self-explanatory—these are some equations which you may find useful. There are often multiple ways to solve a physics problem, so you may not actually need to use all of these equations. It's also important to be clear about what the terms mean. For example, are they talking about a particular force or a resultant force? When a term such as mass appears in an equation, are you using the mass of the correct object? You may also want to think about using the same equation multiple times—does the situation in the problem change over time? Do you need to apply the equation to several different objects or collections of objects? Again, make sure that you substitute correctly into the equation.

Hint 5 - Video

The videos generally show some steps involved in solving the problem with the remainder left for you to complete. In lower levels, particularly Level 1, this may be a nearly complete solution with only the last substitutions not completed. As the levels increase, a smaller part of the solution is shown. It's a good idea to work through the steps shown in the videos yourself rather than just copying down the final state of the video. This will help you to get a better idea of what steps might be useful for similar problems in the future.

Many problems can be solved in multiple ways. Some hint videos only show one method to solve the problem while others will show a couple of methods. Don't be afraid to get started using the method in the video before switching to a different method.

Feedback hints

You may not have noticed, but when you get a question wrong, there will be some feedback underneath the answer box. While this is often just “Please try again”, there is more specific feedback for common wrong answers where we’ve managed to work out what you’ve done wrong. It is always worth reading this feedback, particularly because most of the common wrong answers are the result of a mistake or misconception that’s common to a few different problems.

Sig figs and rounding

Another common problem that students have is with using the correct number of significant figures. The same rule applies across the site, and this is that the correct number of sig figs to use is the lowest number of sig figs in the data that you have used to calculate your answer. You can practise this at `https://isaacphysics.org/gameboards#sig_fig_prac_mastery` . It’s also worth noting that if you use an answer from a previous question to calculate your answer to the new question, you should use the unrounded value of the old answer to avoid accumulating rounding errors. Still give your answer to the same number of significant figures at the end, but don’t round until then.

How to get extra help

If you’re still struggling with some particular questions, you can get in touch with us via Twitter (@isaacphysics) or by using the contact form on our website: `https://isaacphysics.org/contact`. We will generally try to help you understand a particular problem or two, but we won’t teach you a topic or provide answers to the problems. This book came about partly due to people requesting worked solutions to problems to help them understand how to answer particular types of question.

Mechanics

2.1 1D Statics - Jenga Tower

Question

A jenga tower is built consisting of 7 identical blocks, each of mass m, arranged in 4 levels. The bottom two levels contain one block each, the next level up contains two blocks with a one-block gap between them, and the top level consists of 3 blocks stuck together. The tower rests on a table and is stationary. Calculate the magnitude of these forces:

- i the normal force acting upwards on the set of top blocks from just one of the blocks on the layer below.
- ii the normal force acting upwards on one of the two blocks on the second layer down from the block on the layer below.
- iii the normal force acting upwards on the single block on the third layer down from the block on the bottom layer.
- iv the normal force acting upwards on the single block at the bottom of the tower from the table.

Main skills required

This question is an example of a statics problem. This means that, in order to solve it, you are going to need to be confident with drawing free-body force diagrams (diagrams in which only one "body" is drawn and only the forces acting on that body are shown) along with labelled forces acting on the body. You will also need to be able to apply Newton's First Law of Motion, as this describes the forces acting on a body which is stationary, and Newton's Third Law of Motion in order to link the magnitudes of the forces on touching blocks. However as this situation is one-dimensional you only have to deal with forces acting vertically, so you don't need a full understanding of vectors.

Worked Solution

The first thing to do is to draw the whole situation out with the masses of various parts labelled, as shown in Figure 2.1. I went back and labelled the layers with letters after realising it could be hard to refer to a specific layer otherwise. Don't be afraid to add more details to your diagrams later.

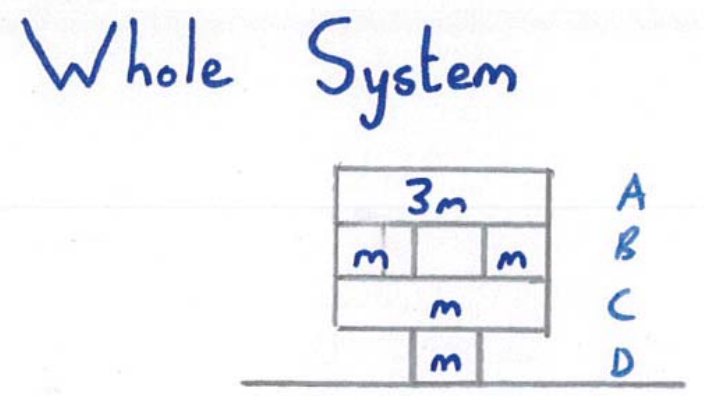
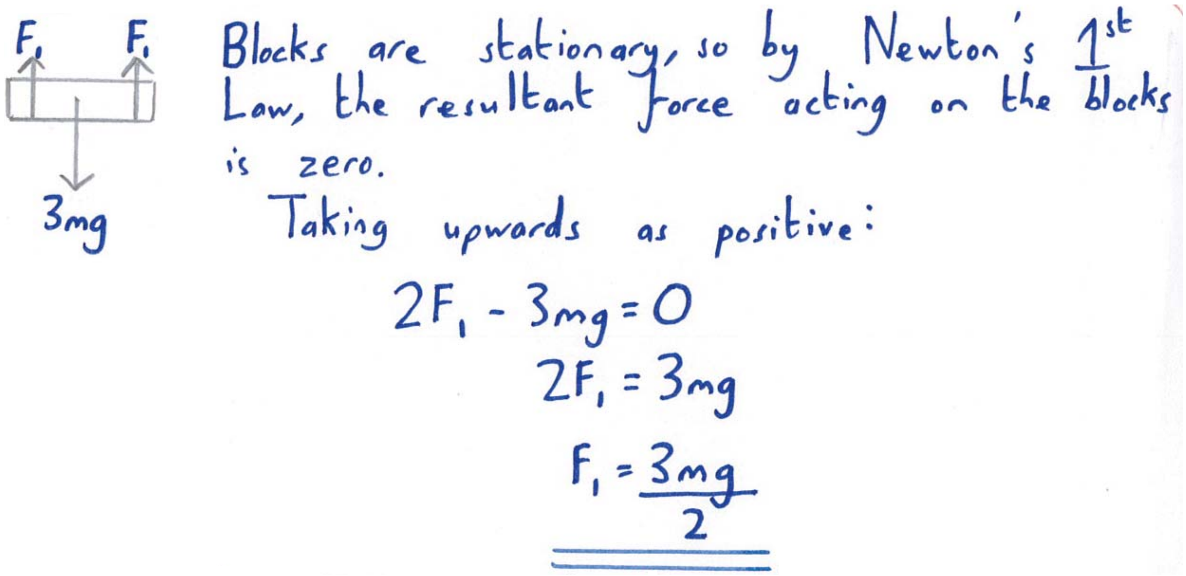


Figure 2.1. The entire Jenga tower, with the top three blocks treated as a single object. The masses of each element are shown, along with labels for each layer

This problem can be solved in a few different ways. One way is to look at the forces on each layer of blocks while another method involves splitting the tower into different sections and only looking at "external" forces acting on that section. I'm going to start with the former and then show how the latter gives the exact same answers (as we should expect!).



Blocks are stationary, so by Newton's 1st Law, the resultant force acting on the blocks is zero.

Taking upwards as positive:

$$2F_1 - 3mg = 0$$

$$2F_1 = 3mg$$

$$\underline{\underline{F_1 = \frac{3mg}{2}}}$$

Figure 2.2 Looking at just the top layer of blocks.

Looking at just the top layer of blocks (A) in Figure 2.2, we need to show all of the forces acting on this layer. We don't need to show any forces between the blocks on the same layer, as when we're considering the entire layer these are "internal" forces and would sum to zero across the entire layer[1]. We do need to include the force due to gravity, of magnitude $3mg$, acting on this layer (A) of blocks and two normal reaction forces from the two blocks below. I've labelled both of these normal reaction forces as F_1 in Figure 2.2 because they must be the same on the two sides, as the situation is symmetric. By saying that "the situation is symmetric", I mean that there's nothing that you can point to in this situation that would allow you to distinguish between the two blocks below the top layer, so they must each exert the same magnitude force on the layer above.

Another thing to notice here is that I've labelled all of the forces (which are vectors) with just their magnitude, leaving it to the direction of the arrow that I've drawn to define their direction. This means that we have to be careful to take into account their directions during the calculations, and if we ever obtain what looks like a negative magnitude, it means that the force is actually acting in the opposite direction to the arrow. In this solution I've chosen to define upwards as positive and don't have to worry about the horizontal direction as there are no components of any of these forces acting horizontally. This allows us to use Newton's 1st Law to solve for the magnitude of the force F_1.

Now we can move on to looking at the second layer (B) in Figure 2.3. Rather than looking at the whole layer, I've chosen to look at just one block, as the situation will be exactly the same for the other block due to the symmetry of the system[2]. It's very important to make sure that all of the forces acting on this block are included. As well as the weight of the block and the normal reaction force from the layer below (which I've labelled as F_2 to make it obvious that it's a force[3]), there's also a normal reaction force from the layer above. This has to exist because of Newton's 3rd Law—as each block on layer B exerts an upwards force on the layer above, the layer above must exert a force downwards (*i.e.* in the opposite direction) on each of the blocks in layer B. These downward reaction forces on layer B must each be of the same magntiude as one of the upward reaction forces on layer A, which is F_1.

All of these forces actually act in a line, but it's important to be able to distinguish the forces in the diagram, so they are all drawn slightly offset. If the question involved rotation, the correct positions would need to be shown more clearly on a larger diagram.

When using Newton's 1st Law to calculate F_2, we had to substitute in for the value of F_1 from the previous part of the question. The reason I didn't immediately use $3mg/2$ here but initially wrote F_1 is so that it's clear in my working where the $3mg/2$ came from. Although it's pretty obvious in this situation, it's a good habit

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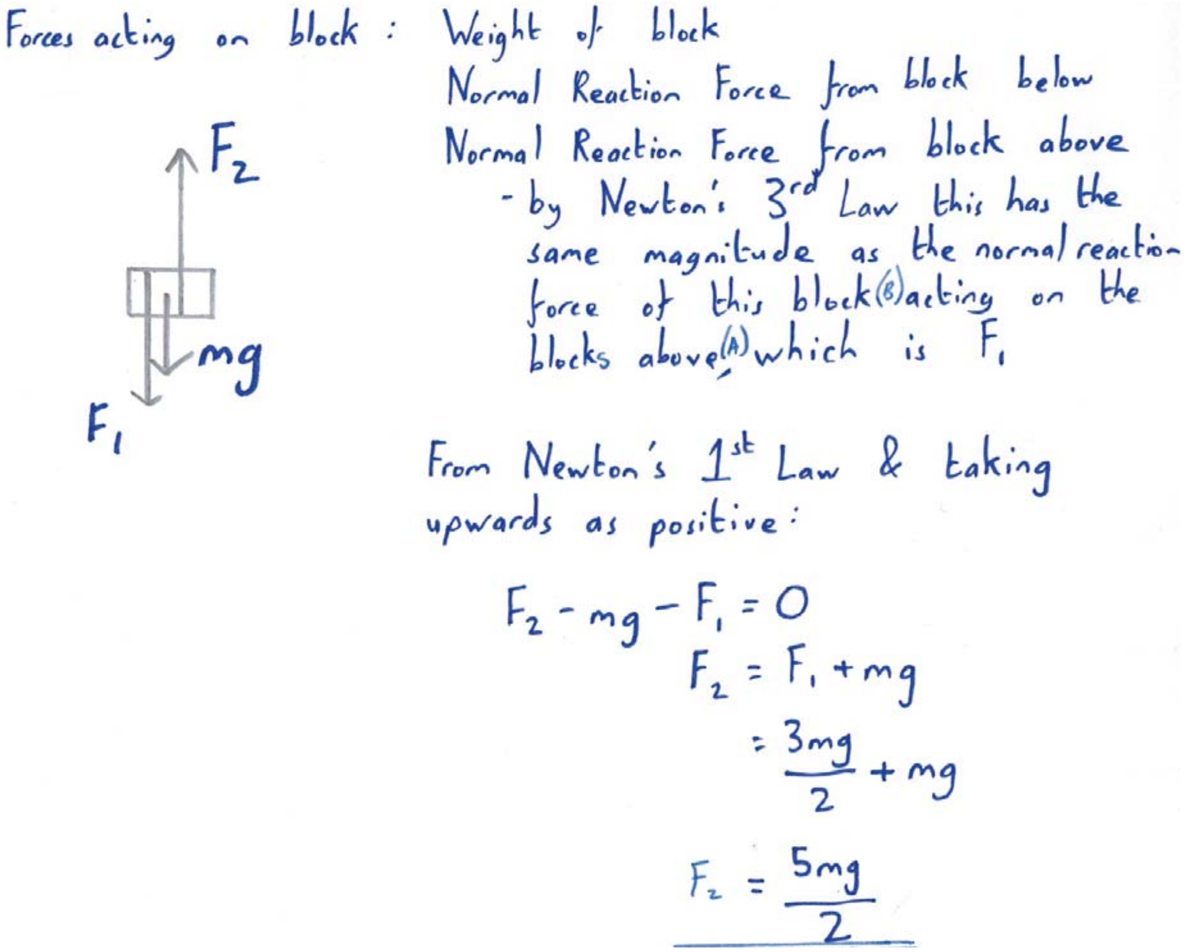
[1] Otherwise the layer would be accelerating sideways, as can be seen from Newton II

[2] Remember that this means that there's nothing in the system which would enable me to distinguish between these two blocks on this level—they are equivalent to each other.

[3] While you can label using whatever letter you want, it's generally a good idea to choose a letter to make it obvious what it is that you're labelling, whether that's a force, a mass, a velocity, *etc*.



Forces acting on block : Weight of block
Normal Reaction Force from block below
Normal Reaction Force from block above
- by Newton's 3rd Law this has the same magnitude as the normal reaction force of this block(B) acting on the blocks above(A) which is F_1

From Newton's 1st Law & taking upwards as positive:

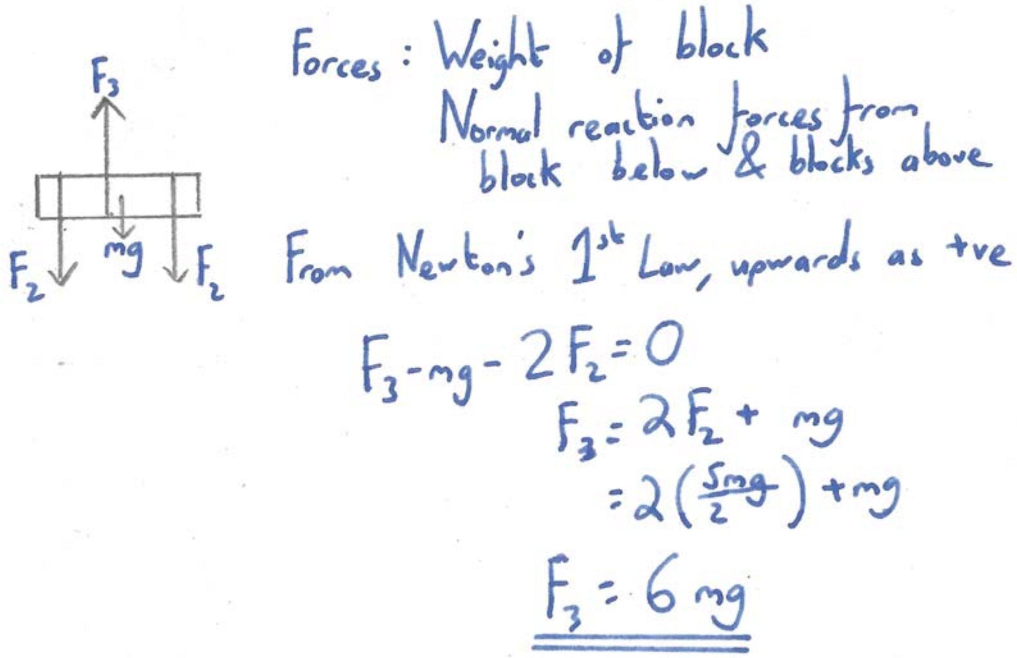
$$F_2 - mg - F_1 = 0$$
$$F_2 = F_1 + mg$$
$$= \frac{3mg}{2} + mg$$
$$\underline{F_2 = \frac{5mg}{2}}$$

Figure 2.3. Looking at one of the two blocks on the layer below the top row (Looking at row B)

to get into—make sure that you can follow your own working when you look back before you move onto more complicated situations.

The rest of this problem can be solved in a similar way, remembering always to include the normal reaction forces from both above and below the current layer - for layer C as shown in Figure 2.4 these normal reaction forces are from other layers of blocks, but for layer D as shown in Figure 2.5, the normal reaction force acting upwards is from the surface the tower is resting upon.

As I said earlier, it's also possible to solve this problem by selecting just some parts of the set of things we're looking at, which we call a "subsystem"—in this case our subsystem is a layer of the tower. By using this method, we can make it so that our subsystem always extends to the top of all the blocks, so we never have to consider a normal reaction force acting downwards, as there is nothing sitting on top of our subsystem of blocks. All of the reaction forces here are labelled the same as in the previous method. We can start with the simplest "subsystem" - the entire tower, as shown in Figure 2.6.

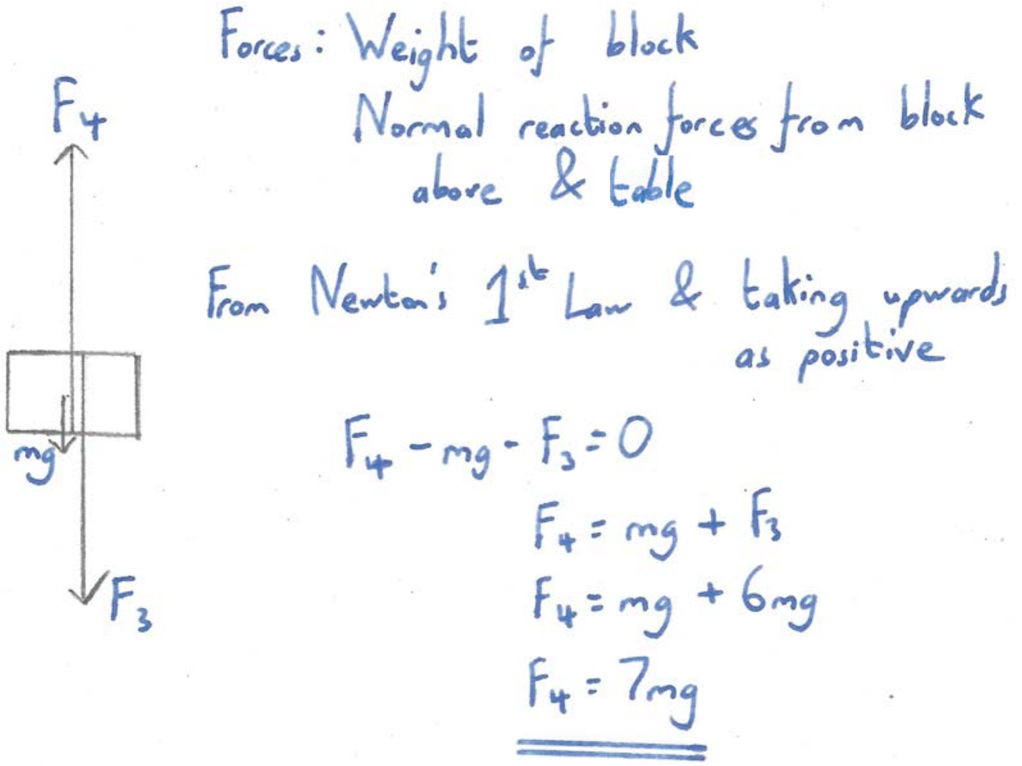


Forces: Weight of block
Normal reaction forces from block below & blocks above

From Newton's 1st Law, upwards as +ve

$$F_3 - mg - 2F_2 = 0$$
$$F_3 = 2F_2 + mg$$
$$= 2\left(\frac{5mg}{2}\right) + mg$$
$$\underline{\underline{F_3 = 6mg}}$$

Figure 2.4. Continuing to solve the rest of the problem by looking at layer C

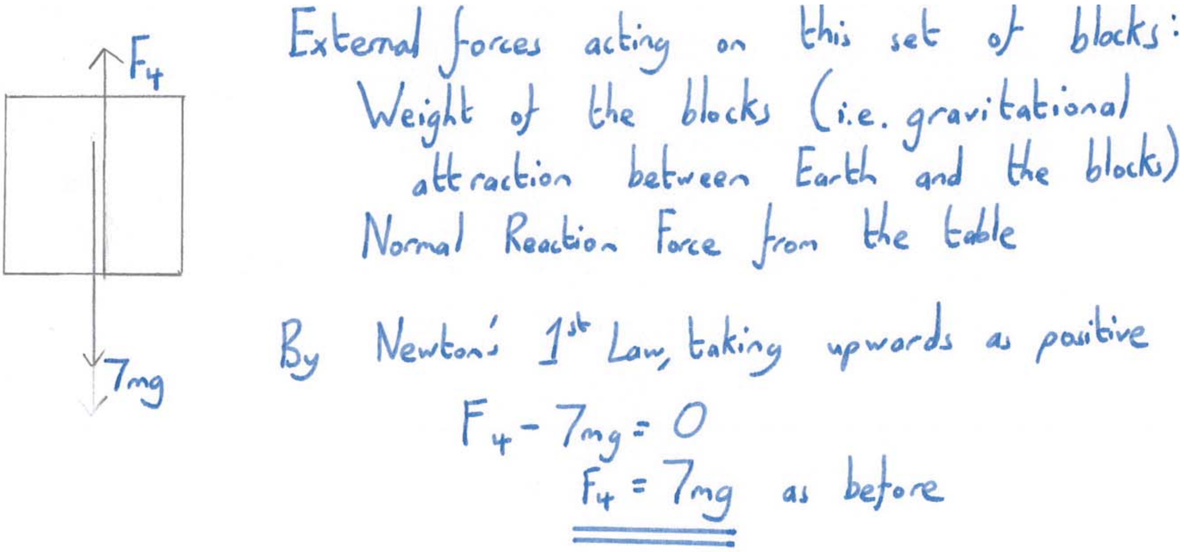


Forces: Weight of block
Normal reaction forces from block above & table

From Newton's 1st Law & taking upwards as positive

$$F_4 - mg - F_3 = 0$$
$$F_4 = mg + F_3$$
$$F_4 = mg + 6mg$$
$$\underline{\underline{F_4 = 7mg}}$$

Figure 2.5. Continuing to solve the rest of the problem by looking at layer D



External forces acting on this set of blocks:
Weight of the blocks (i.e. gravitational attraction between Earth and the blocks)
Normal Reaction Force from the table

By Newton's 1st Law, taking upwards as positive

$$F_4 - 7mg = 0$$

$$\underline{\underline{F_4 = 7mg}} \text{ as before}$$

Figure 2.6 Considering the entire tower as a single system

All of the weight of the blocks included in the subsystem can be thought of as acting through the centre of gravity of the subsystem. We can ignore the normal reaction forces between blocks inside the subsystem because we are treating the subsystem as if it were one object, so the equal and opposite normal reaction forces cancel each other out[4]. It's important to realise that these forces act on different blocks, so normally we'd have to treat them separately and they wouldn't cancel out[5], but since here we're looking at a collection of blocks, these "internal" forces cancel out on the subsystem.

We can then take a "subsystem" consisting of the top three layers of blocks (A, B and C), as shown in Figure 2.7, and a "subsystem" consisting of just the top two layers of blocks (A and B), as shown in Figure 2.8. To finish off the problem we'd then have to take just the top layer of blocks, and this is exactly the same as we've already done in Figure 2.2.

It's up to you which of these two methods you are most comfortable with, but it's worth becoming familiar with both, as some situations are easier to solve with a particular method.

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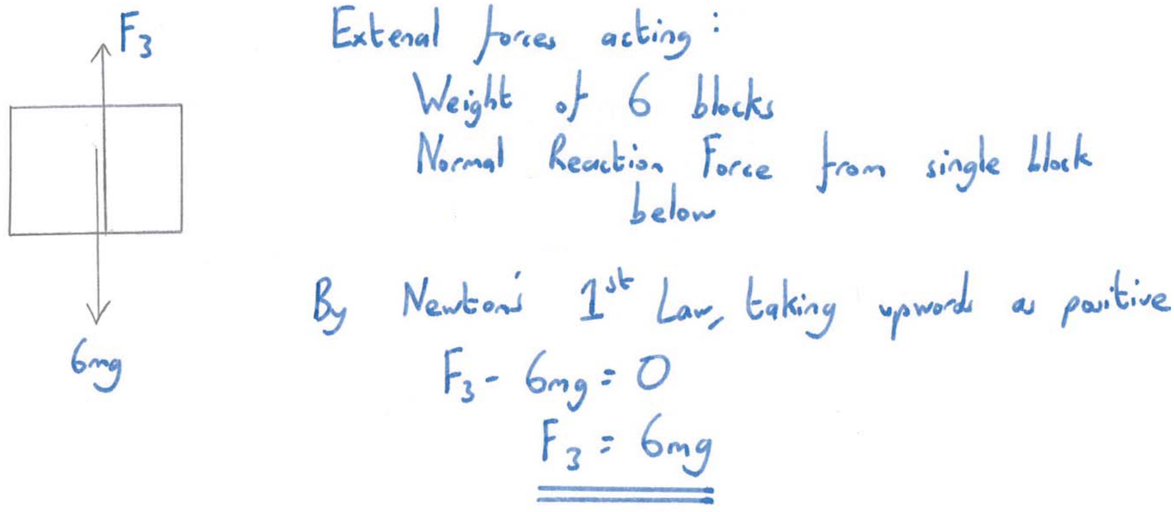
[4] We can use the same argument as we did before with the blocks glued together along with Newton's Third Law to show that these must cancel out.

[5] A common mistake with Newton's Third Law is to think that the "equal and opposite" forces act on the same object and so give zero resultant force. These action-reaction pairs have to act on different objects. If you are balancing out forces on a single object, you are probably using Newton's 1st Law.



External forces acting:

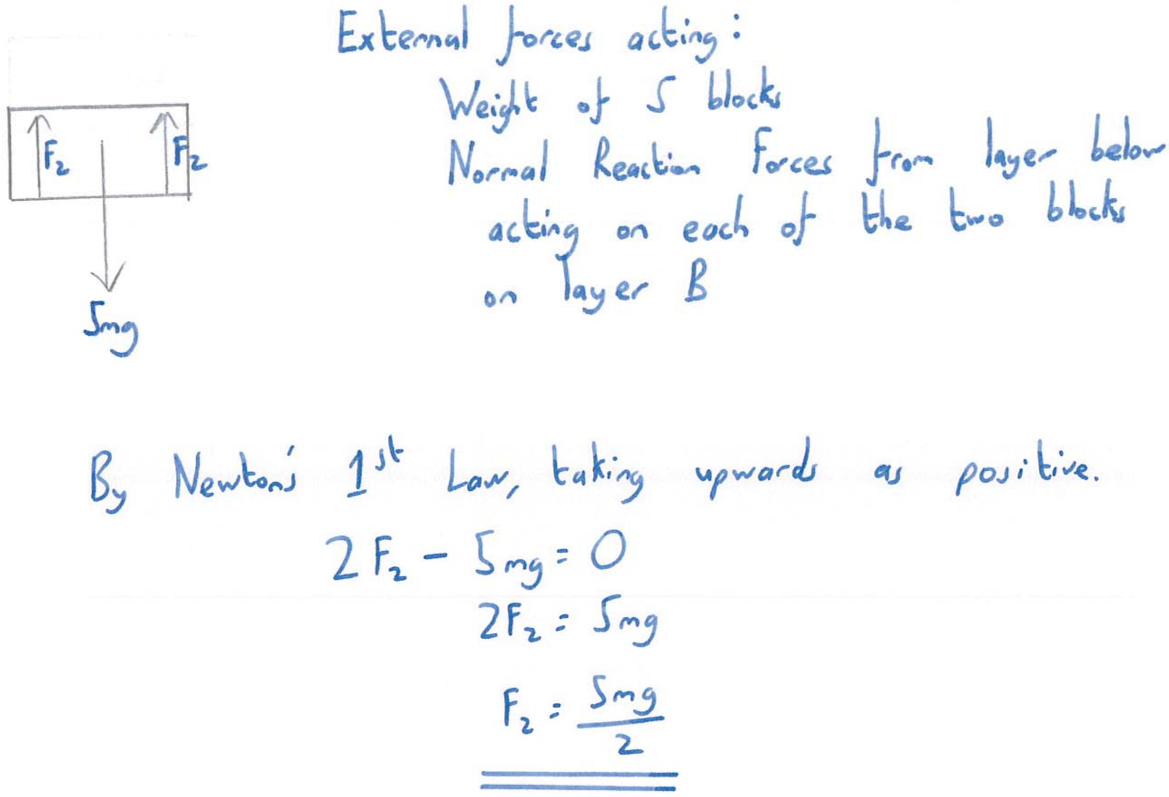
Weight of 6 blocks

Normal Reaction Force from single block below

By Newton's 1st Law, taking upwards as positive

$$F_3 - 6mg = 0$$
$$F_3 = 6mg$$

Figure 2.7 Considering the top three layers as a single subsystem



External forces acting:

Weight of 5 blocks

Normal Reaction Forces from layer below acting on each of the two blocks on layer B

By Newton's 1st Law, taking upwards as positive.

$$2F_2 - 5mg = 0$$
$$2F_2 = 5mg$$
$$F_2 = \frac{5mg}{2}$$

Figure 2.8 Considering the top two layers as a single subsystem

Extra things to think about

All of the previous working assumes that the situation is symmetrical. How would the normal reaction forces change if one of the blocks on layer B has a larger mass than the other, but the sizes and shape of the blocks remain the same?

2.2 2D Statics - Box on a Rough Surface

Question

A box of mass 275 g sits on top of a flat, rough surface. A child attempts to pull it with a horizontal force of 1.2 N, but the box remains stationary. What are the magnitudes of:

- i The normal reaction force from the ground acting on the box?
- ii The frictional force acting on the box?

Main skills required

This question demonstrates the independence of forces acting in perpendicular directions. It uses Newton's 1st Law and the ability to convert between g and kg. A knowledge of what is meant by significant figures is also needed in order to explain why the final answers are as stated.

Worked Solution

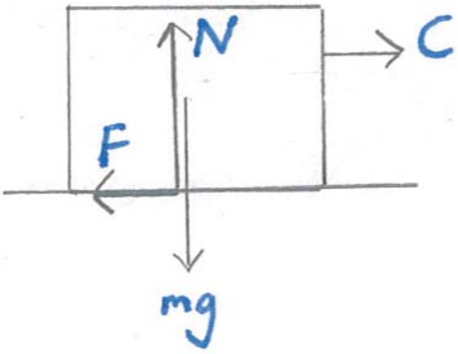
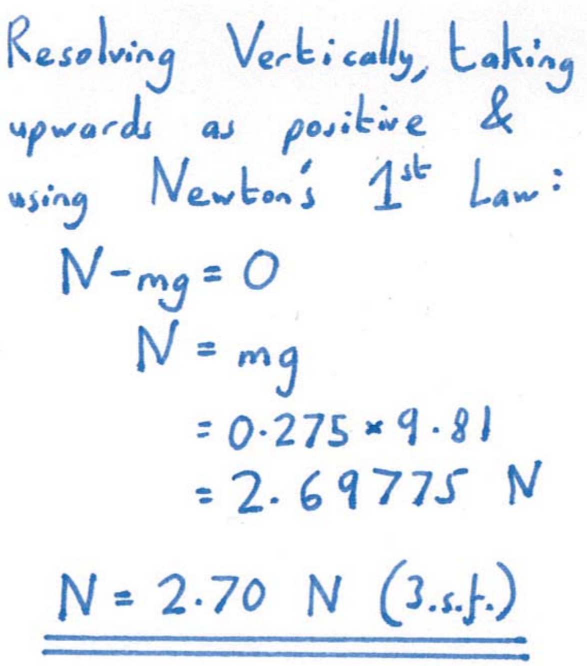


Figure 2.9 The forces acting on the block

All of the given information should first be placed into a diagram to make it all easily accessible. In Figure 2.9 all of this has been represented purely by symbols, rather than the values given. This is to avoid clutter on the diagram while still making it clear what each of these symbols is referring to. The letters used have been chosen to attempt to make it obvious what each letter represents, so F represents a friction force, N the normal reaction force, and C the force from the child. When choosing symbols, you should think about how best to represent all of the pieces of information in the question, which may include forces, velocities, displacements, masses, charges and so on[6]. It's important to be careful about the direction of the frictional force—it acts in a direction so as to oppose the relative motion of the two surfaces[7]. As the child is pulling to the right in my diagram, in the absence of friction this would cause the block to accelerate to the right relative to the ground, so the frictional force acts to the left to oppose this.



Resolving Vertically, taking upwards as positive & using Newton's 1st Law:

$$N - mg = 0$$

$$N = mg$$

$$= 0.275 \times 9.81$$

$$= 2.69775 \text{ N}$$

$$\underline{\underline{N = 2.70 \text{ N } (3.\text{s.f.})}}$$

Figure 2.10 Resolving forces vertically

A good step after this is to think about what physical laws we can use. As the box remains stationary, it is not accelerating, and therefore we can use Newton's First Law to show that there must be no resultant force acting on the box. Rather than try to deal with all of the forces at once, we can instead look at two perpendicular directions; the directions must be perpendicular to ensure that a force in one of those directions has no component acting in the other direction, allowing us to treat them independently. For each of these directions, in this case vertically and horizontally[8], a decision must be made as to in which direction to take the forces as

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[6] Generally speaking, u is used to represent a velocity before some change, and v used to represent the final velocity. You should use subscripts to represent different bodies, so u_1 would be the velocity of object 1 "before", and v_2 would be the velocity of object 2 "after".

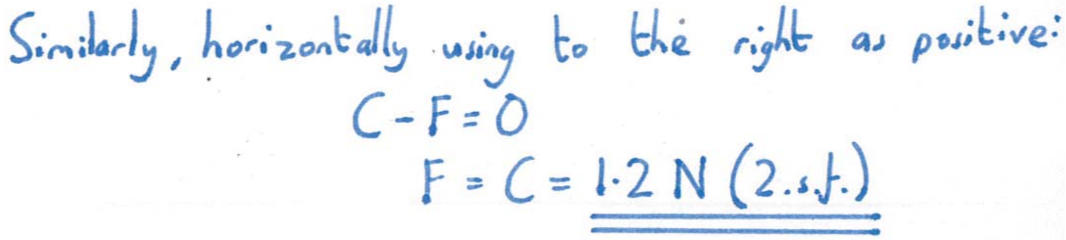
[7] It's particularly important to realise that this is not necessarily against the motion of the overall object. When a car accelerates, the frictional force acting on the wheels from the road is in the direction that the car accelerates—this is how a car can move at all!

[8] While we could have chosen any two perpendicular directions, the fact that the forces acting in this question are all either horizontal or vertical means that this should be the ob-

positive[9].

In Figure 2.10, I've applied Newton's First Law to the vertical components of the forces acting on the box, which in this case is just two vertical forces: the weight of the box and the normal reaction force from the floor. It's important to note that as the mass of the box was given in g, it should first be converted into kg, as the SI base unit of mass is the kilogram (this is the only SI unit where the base unit contains a prefix). If you don't convert the mass, your answer would not be in newtons[10] (as $1\ \text{N} = 1\ \text{kg m s}^{-2}$), and an answer of 2700 N is incorrect.

I can similarly find the frictional force by using Newton's First Law and considering only the horizontal components of the forces, as shown in Figure 2.11.



Similarly, horizontally using to the right as positive:

$$C - F = 0$$

$$F = C = \underline{\underline{1{\cdot}2\ \text{N}\ (2.\text{s.f.})}}$$

Figure 2.11 Resolving forces horizontally

You'll notice that the answer for the normal reaction force was given here to 3 significant figures, as the mass of the box is given to 3 significant figures. The magnitude of the pulling force that the child applies to the box is only given to 2 significant figures, but as this force does not affect the magnitude of the normal reaction force, we don't have to worry about it; however, when we calculate the frictional force, we do use the child's pull, so we can only answer to 2 significant figures.

Extra things to think about

The child stands up, so their pull now acts at some angle to the horizontal. How would the direction of the pulling force affect the normal reaction force from the ground acting on the box and the frictional force needed to stop it from moving?

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vious choice for this problem.

[9] A force in the other direction would then be considered as negative.

[10] While it would be technically correct to give answers to this question in units of g m s^{-2}, physicists would generally try to avoid using this unit. They prefer to use more familiar units made from SI base units.

2.3 2D Statics - Varying Friction

Question

A bag of grain of mass $m = 100$ kg (to 3.s.f.) needs to be moved across a warehouse, but the person pulling it can only apply a horizontal force $P = 450$ N (to 3.s.f.) to move it. A frictional force with a static coefficient of friction of $\mu = 0.55$ opposes this motion. A rope is attached to the top of the bag of grain through a frictionless pulley system so that an upwards force can be applied to the bag. What is the minimum magnitude of this force which would allow the bag to be moved sideways?

Main skills required

This question also uses the independence of forces acting in perpendicular directions. It uses Newton's First Law, as well as the equation for the maximum possible static friction acting between two surfaces, $F_{\text{max}} = \mu N$. It also requires you to think about a "limiting case"—the point at which the behaviour of the system changes sharply.

Worked Solution

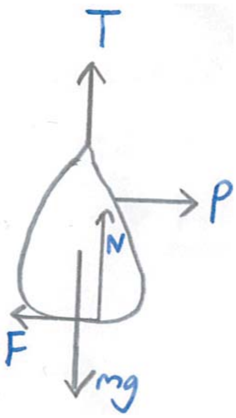
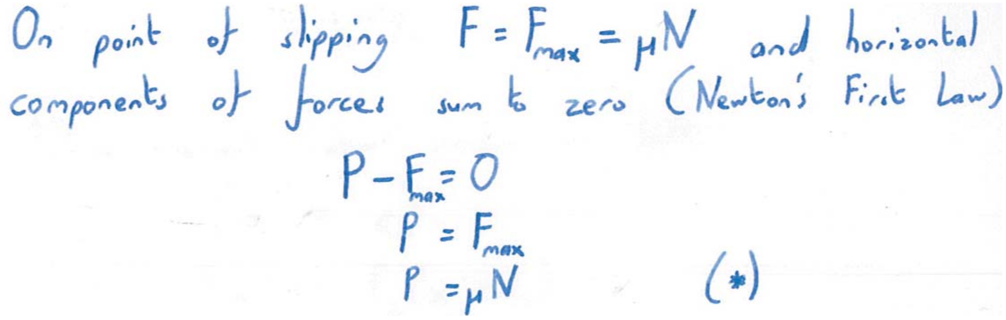


Figure 2.12 The forces acting on the bag

There are actually five forces that you need to consider for this question, so it's important to think carefully about all of these and draw a diagram like that in Figure 2.12 to represent all of these forces acting on the bag. Three of the forces are explicitly described in the question—the upwards force from the rope which we're trying to find (labelled on the diagram as T as this force is a tension[11]), a frictional force (labelled on the diagram as F) and the pulling force P. In addition to these, we also need the weight of the bag, as we assume that this whole situation is taking place on Earth, and the normal reaction force from the floor acting on the bag since they are touching[12].

The next step is to consider what specific situation we care about – it's obvious that if a large enough upwards force were applied to the bag, it would be possible to push it sideways. This upwards force would cause the bag to come off the floor, meaning there was no friction at all between the bag and the floor; however, the question asks for the minimum, so we should be looking at a situation where the pulling force is just big enough to overcome the friction: the "limiting case" where the bag is on the boundary between remaining stationary and moving. At this point:

- The resultant force on the bag is zero (if it was not zero the bag would already be accelerating)
- The frictional force is at its maximum – if the frictional force was not at its maximum, it could increase to counter an increased pulling force.
- Any increase in the upwards force would allow the bag to move sideways, as this would reduce the magnitude of the normal reaction force, causing the maximum frictional force to decrease to lower than the applied pulling force.



On point of slipping $F = F_{max} = \mu N$ and horizontal components of forces sum to zero (Newton's First Law)

$$P - F_{max} = 0$$
$$P = F_{max}$$
$$P = \mu N \qquad (*)$$

Figure 2.13 The horizontal components of forces acting on the bag.

By applying Newton's First Law at this point (to explain why the resultant force must be zero), as shown in Figure 2.13, we can equate the magnitudes of the horizontal forces.

Similarly, as shown in Figure 2.14, we can apply Newton's First Law to the vertical forces in order to work out the normal reaction force, which we need in order to be able to find the magnitude of the frictional force.

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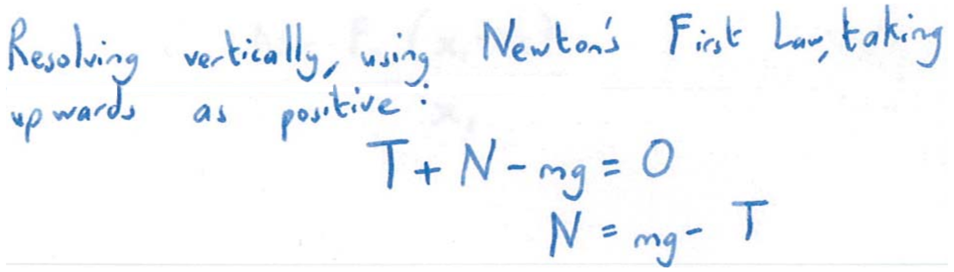
[11] As usual, you could use whatever letter made most sense to you, avoiding those given in the question and any others which may lead to confusion.

[12] I've deliberately drawn the normal reaction force slightly to the side of the weight here. Can you work out why? You may need to think about moments.

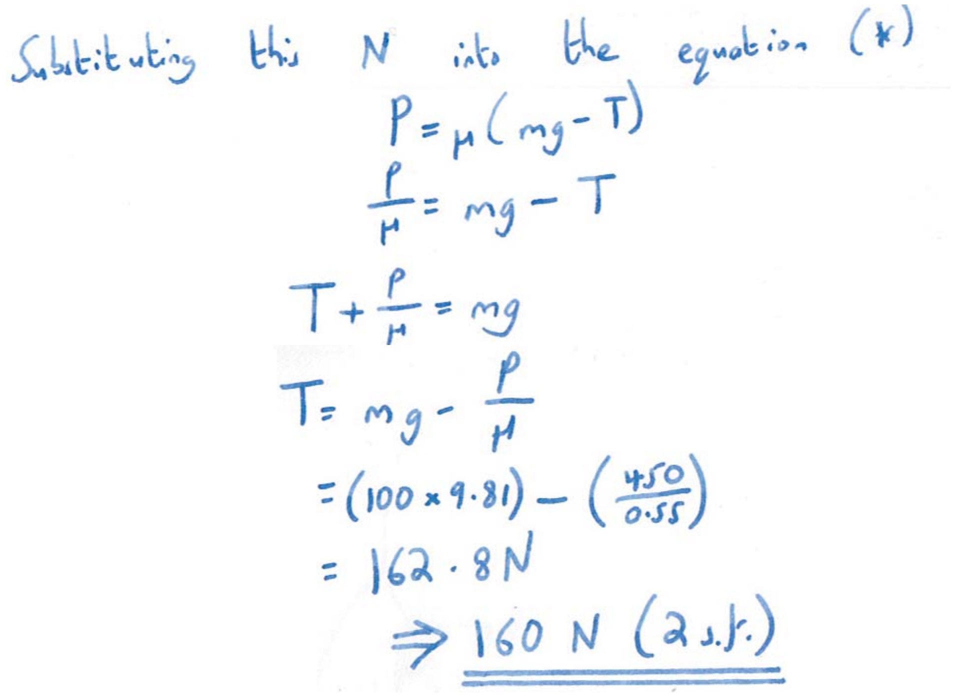


Resolving vertically, using Newton's First Law, taking upwards as positive:

$$T + N - mg = 0$$

$$N = mg - T$$

Figure 2.14 The vertical components of forces acting on the bag.



Substituting this N into the equation (*)

$$P = \mu(mg - T)$$

$$\frac{P}{\mu} = mg - T$$

$$T + \frac{P}{\mu} = mg$$

$$T = mg - \frac{P}{\mu}$$

$$= (100 \times 9.81) - \left(\frac{450}{0.55}\right)$$

$$= 162.8\,N$$

$$\Rightarrow \underline{\underline{160\ N\ (2\ s.f.)}}$$

Figure 2.15 Combining our equations to solve the problem.

Now that we've fully described the relevant physics of this situation, we can combine our equations to get an equation for the magnitude of the upwards force T. Substituting in the values from the question (Figure 2.15) allows us to give a value for this force to only 2 significant figures, as although most of the values are given to 3 significant figures, the coefficient of friction is given to a lower number of significant figures and therefore places a limit on the precision of our answer.

Extra things to think about

How would you solve this problem if, instead of changing the upwards force from the rope, the person pulling the bag could pull at a range of different angles? When the bag starts to move, the friction changes from static friction to dynamic friction which generally has a lower coefficient of friction. What does this say about how the force from the rope could be changed while still allowing the bag to move?